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LETTER TO THE EDITOR

Tunnelling through quantum-dot systems: a study of the magneto-conductance fluctuations

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Abstract. We report on a theoretical study of ballistic transport of electrons through two-probe quantum-dot systems in the tunnelling regime. Large aperiodic conductance fluctuations are observed as a function of the external magnetic field and the electron energy. We analyse the magnetic field correlation functions of the conductance fluctuation for both a stadium-shaped dot and a rectangular dot and find that they are qualitatively different. The correlation function of the stadium-shaped dot agrees well with the semi-classical chaotic-scattering theory, while that of the rectangular dot does not.

Recently, in an interesting experiment, Marcus *et al* [1] measured the conductance of a two-dimensional stadium-shaped quantum dot, connected to the outside by two point contacts, as a function of the magnetic field. The size of the dot was such that transport was in the ballistic regime. Large aperiodic conductance fluctuations were observed at low fields of up to a few thousand Gauss. A resistance peak was also found at zero magnetic field and related to the coherent backscattering of electrons. It is of particular interest to study and understand the origins of the conductance fluctuations observed in submicrometre semiconductor structures such as quantum dots, because such structures are the building blocks of future and current electronic devices [2]. From a theoretical point of view, the semiconductor structures provide, as the experiment of Marcus *et al* showed, a testing ground for theories and ideas in the intriguing field of 'quantum chaos' [3, 4].

Although there is no rigorous and unique definition of quantum chaos, it generally refers to quantum systems whose classical analogue is chaotic, such as a stadium-shaped quantum dot [5, 6], or quantum systems whose eigenvalue spectrum satisfies Dyson ensembles [7], such as the Anderson model. For closed systems, quantum chaos is studied by solving the one-particle Schrödinger equation, and characterizing the statistics of the energy levels [7]. For open systems, such as the devices studied by Marcus *et al*, one is dealing with a problem of scattering of charge carriers by some peculiar boundary, under the influence of an external magnetic field and possibly other effects. In this case much progress and insight have been achieved by studying the statistical properties of conductance fluctuations [8, 9, 10, 11, 12, 13].

In this letter, we report on numerical calculations of the magneto-conductance for a chaotic and a regular structure. We computed the magneto-conductance in the quantum tunnelling as well as in the transmitting regime for the two structures. The chaotic structure considered was a stadium-shaped quantum dot similar to that of Marcus and coworkers [1], while the regular structure studied was in the shape of a rectangle. We focus on the properties of the conductance fluctuation as an external magnetic field is varied. By analysing the correlation function of the fluctuations, we found that these fluctuations show some qualitative differences depending on the structure shape: the fluctuations for the stadium-shaped dot compare reasonably well with the predictions of the semi-classical theory for chaotic scattering, while those of the rectangle do not. These results suggest a novel way of probing chaotic scattering in submicrometre structures.

We focused on the conductance fluctuations in the tunnelling regime for several reasons [14, 15]. First, present understanding of quantum chaos is based on the eigenenergy-level statistics of *closed* billiards. These levels become quasi-bound states when the stadium structure becomes open. In the tunnelling regime the energies of the transmission peaks are closely related to the quasi-bound states, which mediate the resonance transmission. Furthermore, conductance fluctuations are known to arise from the complex scattering of the electron from the boundary and are therefore related to the specific geometry of the structure. It therefore seems important to trap the electron inside the structure for a sufficiently long period of time to reveal the possible chaotic scattering nature of the structure. A simple way to increase the trapping time of the electron is via the use of tunnelling barriers at the openings of the quantum dot. An alternative way of achieving this is to reduce the size of (pinch off) the point contacts of the quantum dot [16].

Before presenting details of our calculations, we briefly summarize the predictions of semi-classical scattering theory. A universal form for the energy correlation function $C(\Delta k)$ of the conductance fluctuations δg for a two-probe chaotic stadium in terms of the the wave vector $k = (2mE/\hbar)^{1/2}$ was previously obtained [13, 17], $C(\Delta k) = C(0)/[1 + (\Delta k/\gamma_{cl})^2]$, where $C(\Delta k) = \langle \delta g(k + \Delta k)\ \delta g(k)\rangle$ averaged over an appropriate k interval and γ_{cl} is a constant. If a measurement is performed using the magnetic field as the tunnelling parameter, semi-classical theory [8, 10] predicts a universal form for the magnetic field correlation function of the conductance fluctuations. For a two-probe chaotic system it gives

$$C(\Delta B) = C(0)/[1 + (\Delta B/\alpha\Phi_0)^2]^2 \tag{1}$$

assuming an exponential distribution $N(A)$ of areas A enclosed by classical trajectories in the structure [8]: i.e., $N(A) \propto \exp(-2\pi\alpha|A|)$ with α a constant. Conductance measurements in the fully quantum regime [1] and a quantum-mechanical calculation of magneto-conductance [8] have been carried out to test these semi-classical formulas. Good agreement was found, even for cases where the mode number of the propagating electron is low. More recently Marcus *et al* measured the conductance fluctuations in the weakly tunnelling regime for a stadium-shaped structure and found that the magnetic field correlation function is similar to that observed in the transmitting regime [18].

The two-dimensional quantum dots we studied are shown later as insets in figure 2. In the ballistic-transport regime it is sufficient to consider the electron scattering from hard confining walls. The whole system, including the leads, is within a uniform external magnetic field B, which points perpendicularly to the x–y plane of the quantum dot. We assume that the electron enters from lead I and exits from lead II. The dimensions of the quantum dots are as follows. Rectangle: width $L_1 = 0.3$ μm, length $L_2 = 0.3788$ μm; stadium: radius 0.15 μm, width $L_1 = 0.3$ μm, and the length of the rectangular part is $L_2 = 0.15$ μm. All lead widths are $W = 0.15$ μm. The width of the potential barrier layer is 150 Å. The barrier height is chosen to be three times the incident electron energy. Note that the areas of the two dots are the same.

To calculate the transmission coefficient of a single electron through the structure, we used a finite-element boundary-matching scheme [19, 20]. The system was divided into two parts: the leads, and the dot (including the barriers). The Schrödinger equation in the dot region was solved using a finite-element triangular discretization [19, 20], while in the leads it was solved using appropriate combinations of Kummer functions [21]. The wave functions and their spatial derivatives were then matched at the boundaries. This procedure gave the transmission and reflection coefficients, which were then used to obtain the conductance using the Landauer formula [22]. The method also gives the wave function everywhere in the system.

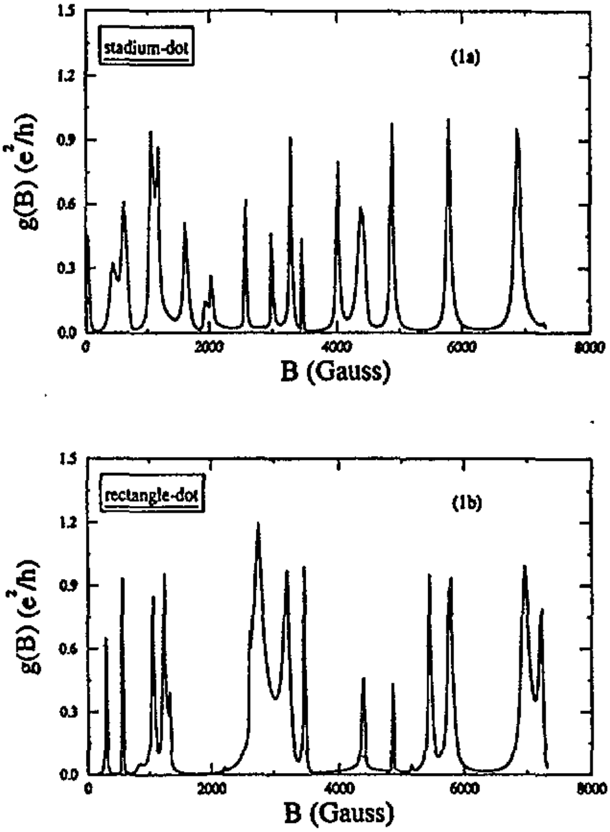

Figure 1. The conductance $g(B)$ versus the magnetic field B for (a) the stadium-shaped dot and (b) the rectangular dot.

As mentioned above, the transmission in the tunnelling regime is of resonance nature. These resonances are mediated by the quasi-bound states inside the quantum dots. They become bound states when the quantum dots are closed [23]. These quasi-bound-state energies must be related to the intrinsic nature of the level statistics of the corresponding closed system. It is well known that for a regular structure such as the rectangular dot, the energy level spacings satisfy Poisson statistics [3] where the probability of finding levels of close energy is high. For a classically chaotic stadium-shaped dot, the spacings satisfy a Wigner distribution [3]. Indeed, we found qualitatively similar behaviour in the open system: for the rectangular dot some conductance peaks are extremely close to each other and many overlap substantially, which indicates the near-degeneracy or degeneracy of the energy levels of the corresponding closed system. For the stadium-shaped dot, the peaks are more separated from each other due to the lack of degeneracy among the energy levels of the closed system. Thus, in the tunnelling regime, the behaviour of the conductance is related to the level statistics of the closed system [24], as expected. On the other hand, as the system is made more transmissive by removing the tunnelling barriers, the resonant peaks merge, making it difficult to make a direct connection between the levels of the open and closed systems.

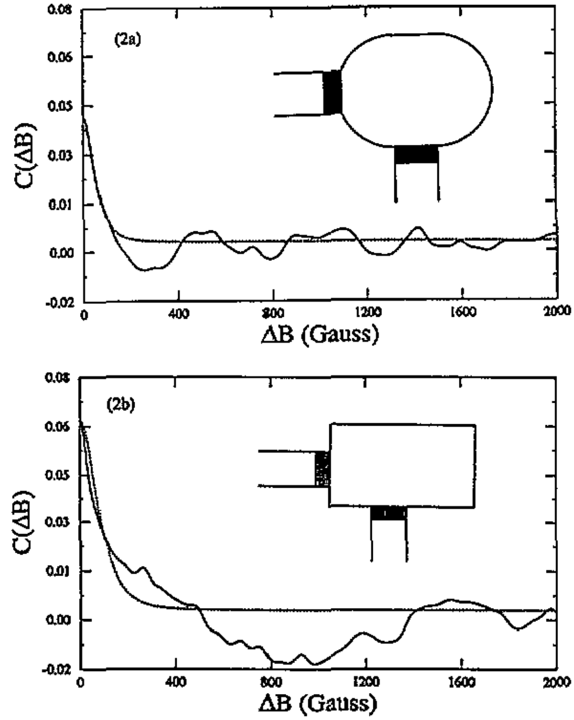

Figure 2. The magnetic field correlation function $C(\Delta B)$ versus magnetic field ΔB for (a) the stadium-shaped dot and (b) the rectangular dot (see the insets). Here the solid lines are from our calculation and dotted lines correspond to the semi-classical theory. The dimensions of the dots are as follows. Stadium: radius $R = 0.15$ μm, width $L_1 = 2R = 0.3$ μm, and the length of the rectangular part is $L_2 = 0.15$ μm; rectangle: width $L_1 = 0.3$ μm, length $L_2 = 0.3788$ μm. All lead widths are $W = 0.15$ μm. The width of the potential barrier layer (shaded region) is 150 Å.

Experimentally it is easier to use the magnetic field as a control parameter. Figure 1 shows the conductance of the dots as a function of the magnetic field B. We have fixed the electron energy to be just above the third zero-field subband energy of the leads, $kW = 9.5$. Similar to the findings of Marcus *et al*, large aperiodic conductance fluctuations are observed for both the rectangular and the stadium-shaped structures. To study the statistics of the conductance fluctuations we have calculated the magnetic field correlation function $C(\Delta B) = \langle \delta g(B + \Delta B)\ \delta g(B)\rangle$. The results are shown in figure 2(a) for the stadium-shaped dot and figure 2(b) for the rectangular dot. Conductance fluctuations $\delta g(B)$ were extracted from $g(B)$ by subtracting a smoothed average of $g(B)$. The correlation functions predicted by the semi-classical theory (1) are shown as the dotted lines in figure 2. Figure 2(b) shows that data for the regular structure, i.e., the rectangular dot, do not agree with the semi-classical prediction. However, for the stadium-shaped dot, our data fit reasonably well with the prediction for a range of ΔB values, as shown in figure 2(a). The oscillation around the semi-classical result in the tail of figure 2(a) can be understood as originating from non-universal interference effects [8]. This suggests that, at least in the tunnelling regime, the universal signature of 'quantum chaos' can be detected by measuring the conductance fluctuation. Moreover, the statistic properties of the conductance fluctuations behave quite similarly to the results predicated by the semi-classical theory. This is also consistent with the experimental observation [18].

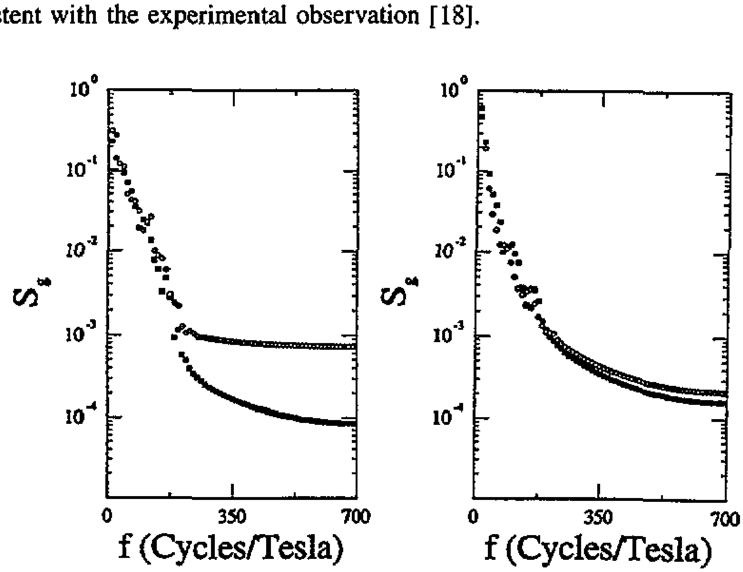

Figure 3. The averaged power spectrum of the conductance fluctuation $S_g(f)$ versus magnetic 'frequency' f for the stadium-shaped dot (solid squares) and the rectangular dot (open circles). For comparison, we plot $S_g(f)$ versus f for the situations where tunnelling barriers are present, in the left panel, and where the barriers are removed, in the right panel.

It is useful to examine the averaged power spectrum of the conductance fluctuation $S_g(f)$ by taking a Fourier transform of the magnetic field correlation function. Here f is the magnetic 'frequency' in units of cycles T^{-1}. This helps to reduce the non-universal low-frequency contribution to the correlation function [8]. The left panel in figure 3 shows $S_g(f)$ for the stadium-shaped dot (solid squares) and the rectangular dot (open circles). Again we find that the sets of data behave differently, consistently with the behaviour observed experimentally [1], despite the fact that the experiment was performed in the transmitting regime. For the latter case this behaviour has a good explanation in terms of semi-classical theory, as discussed in [1, 12, 9].

When the tunnelling barriers are removed, the systems become quite transmissive because of the relatively large probe width chosen (as compared with the quantum-dot dimensions). As mentioned above, in this case the resonant nature of the transmission is less directly related to the eigenenergies of the corresponding closed systems. The openness of the structure gives a much shorter trapping time for the electron. Therefore, in this situation, we do not expect chaotic scattering of the electron in the stadium-shaped dot to play an important role. The right panel of figure 3 shows the averaged power spectrum when no tunnelling barriers are present. Clearly, the differences between the two sets of data are much smaller than those in the tunnelling regime.

In summary we have studied magneto-conductance for both rectangular and stadium-shaped quantum dots in both the presence and the absence of tunnelling barriers at the leads. In the tunnelling regime, the eigenenergies and the statistical properties of the conductance fluctuations for the open and closed systems are related. Similar to experimental measurements, we observed large aperiodic conductance fluctuations for both structures at low magnetic fields. However, these fluctuations give rise to quite different correlation functions. For the stadium-shaped quantum dot the correlation function agrees well with the semi-classical chaotic-scattering theory, while for the rectangular dot there is no agreement. Without the barriers, the openness of our structures makes chaotic scattering non-essential. In this case the averaged power spectra of the two structures are quite close to each other. Thus, at least in the tunnelling regime, the statistical properties of the conductance fluctuation of the classically chaotic stadium-shaped structure can be detected and the importance of chaotic scattering studied. Finally we note that the tunnelling regime has no classical analogue, and thus the applicability of the semi-classical theory is questionable. However, given that our data seem to confirm the semi-classical predictions in this regime, further theoretical investigation is needed to clarify this situation.

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